UNIVERSITY OF SASKA

DEPARTMENT OF MATHEMATICS

Time: 3 hours MATH 225.3 FINAL EXAMINATIO

CLOSED BOOK - NO NOTES, NO TEXTS, NO CALCULATORS

Values

[2] 1. (a) How do you find the area of a parallelogram determined by $\vec{a}$ and $\vec{b}$?

[2] (b) How do you find the volume of the paralleliped determined by $\vec{a}$ and $\vec{b}$ and $\vec{c}$?

[2] (c) How do you find the angle between two intersecting planes ?

[6] (d) If $\vec{u}$ and $\vec{v}$ are differentiable vector functions, c is a scalar, and f is scalar function, write the rules for differentiating

(i) $\vec{u}(t) + \vec{v}(t)$ (ii) $c\vec{u}(t)$ (iii) $f(t)\ \vec{u}(t)$

(iv) $\vec{u}(t) \cdot \vec{v}(t)$ (v) $\vec{u}(t) \times \vec{v}(t)$ (vi) $\vec{u}(f(t))$

[4] (e) (i) What is a smooth curve ?
(ii) How do you find a tangent vector at a point of a smooth curve ?
(iii) How do you find the tangent line ?
(iv) How do you find a unit tangent vector ?

[3] (f) (i) Define the linearization of f at (a, b, c).
(ii) What is the linear approximation ?
(iii) What is the geometric interpretation of the linear approximation ?

[4] (g) Explain how the Lagrange multiplier method works in finding the extreme values of f(x, y, z) subject to the constraint g(x, y, z) = k .

[2] (h) Write an expression for the area of a surface with equation $x = f(y, z)$, $(y, z) \in D$.

2. The temperature at a point (x, y, z) is given by

$$T(x,y,z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in °C and x, y, z in meters.

[6] (a) Find the rate of change of temperature at the point $P(2,-1,2)$ in the direction toward the point $(3,-3,3)$.

[2] (b) In which direction does the temperature increase fastest at P?

[2] (c) Find the maximum rate of temperature increase at P.

[4] 3. (a) Find an equation for the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 12$ at the point $(\sqrt{2}, \sqrt{2}, \sqrt{2})$.

[6] (b) Find the points, if any, on the ellipsoid $2x^2 + 3y^2 + z^2 = 12$ where the tangent plane is parallel to the plane $4x - 3y + z = 1$.

[10] 4. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $2x + 3y + z = 6$.

[8] 5. Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy$ by reversing the order of integration.

[6] 6. (a) Sketch the graphs of $r = 3\cos\theta$ and $r = 1 + \cos\theta$ and find the points where the graphs intersect.

[8] (b) Find the area of the region inside $r = 3\cos\theta$ and outside $r = 1 + \cos\theta$.

[7] 7. Find the work done by the force field

$$F(x,y) = \left(\frac{1}{x+y^2}\right) i + ye^x j$$

in moving an object along the curve with vector equation $r(t) = 4t^2 i + tj$, $1 \le t \le 3$.

[8] 8. Find the area of the surface $z = xy$ which lies in the first octant and within the cylinder $x^2 + y^2 = 1$.

[8] 9. Let S denote the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = 3x$, $y = 0$, $z = 0$ in the first octant. Sketch a picture of S and find its volume.

[100]